WE2F-1

Equivalent circuit of radiating longitudinal slots in dielectric filled rectangular waveguides obtained with FDTD method

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***Abstract* – A rigorous FDTD characterization of a longitudinal radiating slot in a dielectric filled rectangular waveguide in terms of an equivalent shunt admittance is presented. The FDTD method allows one to account for a number of details that affect the antenna performance, such as the waveguide wall thickness, the presence of a finite or infinite flange, a dielectric layer over the slot. High numerical efficiency has been obtained by using Stegen's factorization [4] of the slot admittance. Comparison with other methods (MoM) and with experimental results have shown very good agreement with FDTD simulations even in the computation of the resonant length, which is the most critical parameter. The proposed equivalent circuit allows for a fast and accurate analysis of a radiating slot in a frequency range of the order of 15%.**

I. INTRODUCTION

Because of its good efficiency and relatively low cost, slotted waveguide technology is a promising candidate for the fabrication of electronically scanning antennas for advanced communication services. Such antennas can typically be used for mobile satellite terminals.

For practical implementation of the beam steering, however, the problem of grating lobes must be solved. Using a standard air-filled waveguide the beam steering is limited to only narrow angles, with a maximum scanning angle between 6° + 25°. A simple way to achieve wider scanning angles is to reduce the waveguide width by filling the waveguides with a dielectric material, thus reducing the distance between radiating elements [7].

For the design of a slotted dielectric filled waveguide array the resonant length of each slot must be known very accurately. The slot is by itself narrow-band and the bandwidth of the array is further reduced by the mutual coupling between the slots. By modelling each slot by an equivalent shunt admittance [1]-[3] large planar arrays of slots can be designed in an efficient manner, accounting also for the mutual coupling effects.

In this paper the FDTD method has been used for an accurate computation of the equivalent circuit of an isolated longitudinal slot cut in the broad wall of a dielectric filled rectangular waveguide, while a proper factorization of the slot admittance has been used for high numerical efficiency.

By using the FDTD method, the waveguide wall thickness, the presence of a finite or infinite flange and a dielectric layer over the slot can simply be accounted for, making this method very suited for this kind of applications.

II. EQUIVALENT MODEL OF THE RADIATING SLOT

Large arrays with hundreds of slots can be designed in an efficient manner using Elliott's method of design [1]-[3], where a symmetrical standing wave is assumed as the voltage distribution along each slot. A detailed analysis shows that if the slot is near resonance this assumption is a very good approximation [2], [3]. This implies that the scattering due to a longitudinal slot in the broad wall of a rectangular waveguide is symmetrical and its equivalent circuit simply consists of the shunt admittance $Y(s,l,f)$, depending on the slot offset s, the slot length l and frequency (Fig. 1). The slot width is set to a fixed quantity, usually between $l/10$ and $l/20$, depending on bandwidth requirements.

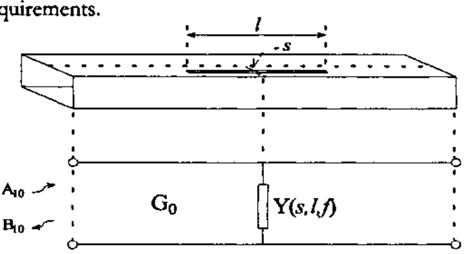

Fig. 1. Longitudinal slot in the broad wall of a rectangular waveguide and its equivalent circuit.

In this paper, the FDTD method has been adopted to rigorously compute the scattering parameters of a slot; the slot admittance of fig.1 is then evaluated from the scattering matrix.

The scattering parameters of the circuit shown in Fig. 1 are:

$$S_{11} = S_{22} = -\frac{Y/G_0}{2+Y/G_0} \tag{1}$$

$$S_{12} = S_{21} = 1 + S_{11} \tag{2}$$

and G_0 is the characteristic admittance of the waveguide.

Using (1) and (2) the normalized admittance can be expressed in terms of the reflection coefficient S_{11} (back-scattering definition):

$$\frac{Y(s,l,f)}{G_0} = \frac{-2S_{11}}{1+S_{11}} \tag{3}$$

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or in terms of the transmission coefficient S_{21} (forward-scattering definition):

$$\frac{Y(s,l,f)}{G_0} = 2\left(\frac{1}{S_{21}} - 1\right) \quad (4)$$

Since symmetrical scattering is an approximation, the equation (2) is not satisfied exactly, so that the results obtained through (3) and (4) show a small difference.
Another possible expression for Y/G_0 is:

$$\frac{Y(s,l,f)}{G_0} = 2 \cdot \frac{S_{11} + S_{21} - 1}{S_{11} + S_{21} + 1} \quad (5)$$

This equation derives from (3) with S_{11} substituted by the average between the computed $S_{11,FDTD}$ and the reflection coefficient derived through (2) from computed $S_{21,FDTD}$:

$$S_{11} = \frac{S_{11,FDTD} + S_{21,FDTD} - 1}{2} \quad (6)$$

It can be shown that expression (5) gives a better approximation of the slot behavior.

Since accurate FDTD simulations are rather CPU intensive, for the suitability of the method it is essential to minimize the computing effort. To this purpose, we have used for Y/G_0 the factorisation proposed by Stegen [1], [4]:

$$\frac{Y(s,y)}{G_0} = \frac{G_r}{G_0}\frac{G + jB}{G_r} = g(s)h(y) = g(s)\left[h_1(y) + jh_2(y)\right] \quad (7)$$

where:

s is the offset of the slot, l is the length of the slot,

$g(s) = \frac{G_r(s)}{G_0}$ is the normalized resonant conductance,

$y = \frac{l}{l_r(s,f)}$ is the ratio of length to resonant length,

$l_r(s,f) = \frac{\lambda}{2\pi}v(s) = \frac{c_0}{2\pi f}v(s)$ is the resonant length. (8)

$h(y) = h_1(y) + jh_2(y) = \frac{G + jB}{G_r}$ is the ratio of the slot admittance to the resonant conductance.

With the factorization (7), the normalized slot admittance is approximated as the product of the normalized resonant conductance (which is a real function of only the slot offset s) times the ratio between the admittance and the resonant conductance, which is a function of l/lr. The resonance condition is verified when Y/G_0 is purely real; in this case we have: $l = l_r$, $G = G_r$, and $Y/G_0 = g$.

In his work [4] Stegen experimentally shows that the resonant length of a slot normalized to the free space wavelength is a function only of the offset s, as shown in equation (8). In this manner, the computation of the equivalent slot admittance, which is a function of the variables s, l and f is reduced to the computation of three separate functions of a single variable: $g(s)$, $v(s)$, $h(y)$.

We have used the FDTD method to compute the scattering parameter S_{11} as a function of frequency for a set of offsets $\{s_i\}$ in the range $0 \div 0.35a$ and for a fixed slot length $\bar{l}$ close to the resonant length at f_0. The resonant length is significantly affected by the permittivity of the dielectric filling the guide. A first guess for the resonant length can be chosen as

$$\bar{l} = \lambda_0 / \sqrt{2(\varepsilon_r + 1)} \quad (9)$$

The i-th simulation returns a function $S_{11,i}(f)$ in a band of frequency, since a modulated gaussian pulse excitation has been used for the incident TE_{10} mode. For each simulation with a different s_i there is a frequency $f_{r,i}$ for which $S_{11,i}(f)$ is real and negative. At that frequency the slot is resonant and its length $\bar{l}$ is defined as the resonant length.

We can thus compute the values of the functions $v(s)$ and $g(s)$ for each offset s_i of the set, using equations (3) and (8) :

$$v(s_i) = \frac{2\pi \bar{l} f_{r,i}}{c_0} \quad (10)$$

$$g(s_i) = \frac{2\left|S_{11,i}(f_{r,i})\right|}{1 - \left|S_{11,i}(f_{r,i})\right|} \quad (11)$$

A polynomial interpolation has then been applied to approximate $v(s)$ and $g(s)$ in a continuum.

While these two functions describe the slot at resonance, the function $h(y)$ describes its behaviour off-resonance. Note that $h(y)$ does not depend on the offset. To verify the accuracy of this assumption, for each offset s_i a different function $h_i(y)$ has been computed.

The $h_i(y)$'s are defined in terms of the ratio y between the slot length and its resonant length at the centre frequency:

$$y = \frac{l}{l_r(f_0)}$$

Following the same approach adopted to compute the functions $v(s)$ and $g(s)$, for each offset s_i a new set of simulations for different lengths should be performed.

From equation (10), the product between a certain frequency and the resonant length at that frequency can be considered as a constant in a narrow band:

$$v(s_i) = \frac{2\pi}{c_0}\bar{l} f_{r,i} = \frac{2\pi}{c_0} l_r(f_y) f_y \quad (12)$$

It is therefore possible to vary the ratio y by changing the frequency instead of the length as follows:

$$y = \frac{\bar{l}}{l_r(f_y)} = \frac{f_y}{f_{r,i}} \quad \Rightarrow \quad f_y = y f_{r,i} \quad (13)$$

For each offset, the function $h_i(y)$ is thus computed for y in the range 0.9 ÷ 1.1:

$$h_i(y) = \frac{-2S_{11,i}(f_y)}{[1 + S_{11,i}(f_y)]g(s_i)} \quad (14)$$

where the scattering coefficients $S_{11,i}(f)$'s are the same already computed for the functions $v(s)$ and $g(s)$.

III. Results

To verify the accuracy of the method, a comparison with experimental and simulated results has been performed for a longitudinal slot (slot width w=1.5875 mm) in a WR90 at the frequency of 9.375 GHz. These results are in excellent agreement with those obtained with Method of Moments (MoM) by Josefsson [5] and with experimental results, even for the resonant length, which is the most critical parameter.

As can be seen from Figs. 2 and 3, all functions $h_i(y)$ are very close together; this confirms the good approximation of the variables separation (7). In this work, $h(y)$ has been chosen as the average of the $h_i(y)$'s. Fig. 4 shows the normalized resonant conductance versus the offset.

The functions $h_1(y)$, $h_2(y)$ and $g(s)$ are in good agreement with the experimental results obtained by Stegen [4].

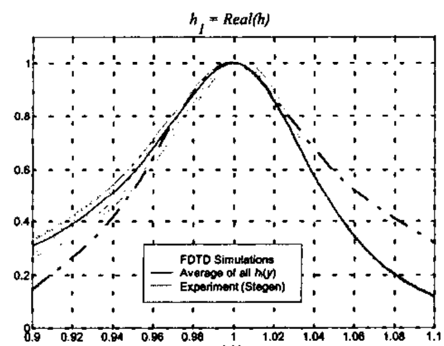

Fig. 2. G/G$_r$ vs. l/l_r for a longitudinal slot in a WR90 at 9.375 GHz.

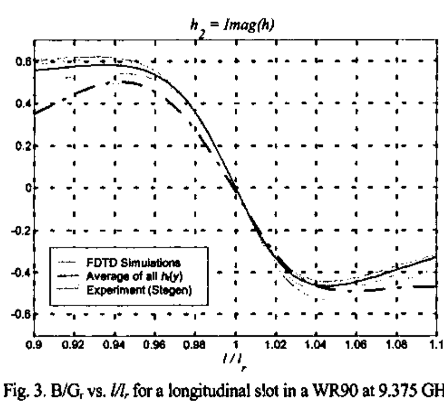

Fig. 3. B/G$_r$ vs. l/l_r for a longitudinal slot in a WR90 at 9.375 GHz.

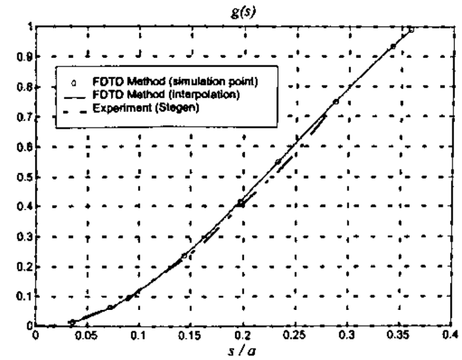

Fig. 4. Normalized resonant conductance versus normalized offset, for a longitudinal slot in a WR90 standard, at 9.375 GHz.

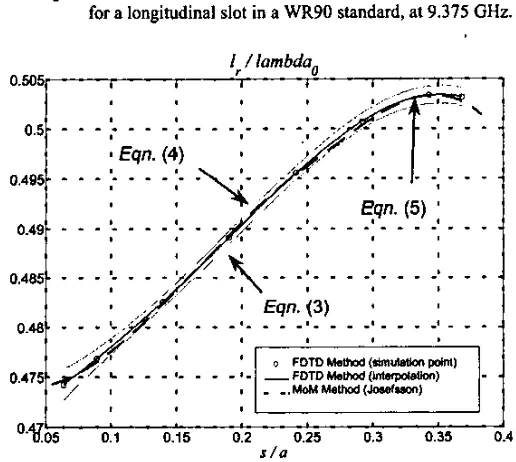

Fig. 5. Resonant length versus offset obtained with three different definitions (back.scattering, forward-scattering, intermediate) for a longitudinal slot in a WR90 at 9.375 GHz.

Fig. 5 shows the normalized resonant length versus the offset, computed using the three different definitions of the equivalent admittance, expressed by equations (3-5). These results are compared with a simulated result obtained by Joseffson with the MoM [5], and the best agreement is obtained using equation (5).

The method has then been used to analyse a longitudinal slot (slot width w=1.5 mm) cut in the broad wall of a WR90 waveguide (a = 22.86 mm, b = 10.16 mm, t = 1.59 mm), filled with a dielectric material with ε_r = 2.1. The centre frequency has been chosen at 6.7 GHz, which is in the range of minimum waveguide loss.

Figs. 6 and 7 show the behaviour of $h(y)$ versus the normalised slot length l/l_r. The comparison with Figs. 2 and 3 show that, as it could be expected, the dielectric filling has also the effect of narrowing the slot bandwidth.

The normalised resonant conductance $g=G_r/G_0$ vs. the slot offset is plotted in Fig. 8. It is interesting to note, by comparison with Fig. 4, that because of the dielectric filling, the slot admittance exhibits a stronger dependence on the offset. As an example, the slot conductance G_r equals the waveguide admittance G_0 for s/a=0.36 for the empty waveguide and for s/a=0.27 for the filled waveguide.

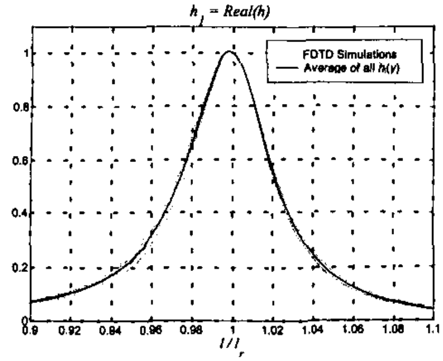

Fig. 6. G/G$_r$ vs. l/l_r for a longitudinal slot in a dielectric filled WR90 at 6.7 GHz.

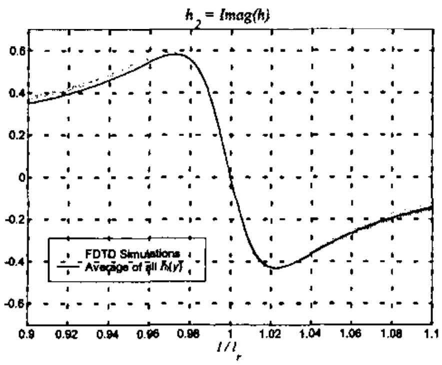

Fig. 7. B/G$_r$ vs. l/l_r for a longitudinal slot in a dielectric filled WR90 at 6.7 GHz.

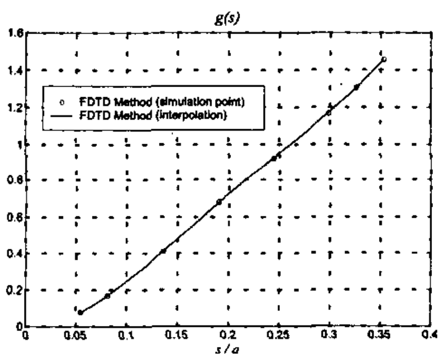

Fig. 8. Normalized resonant conductance versus normalized offset for a longitudinal slot in a dielectric filled WR90 at 6.7 GHz.

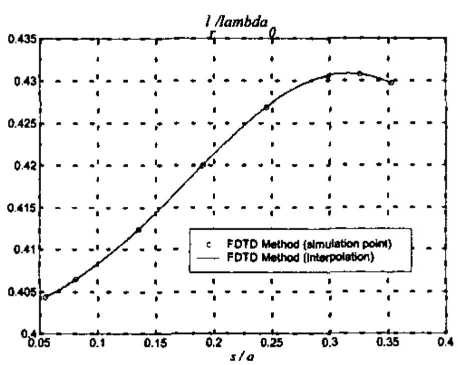

Fig. 9. Normalized resonant length versus normalized offset for a longitudinal slot in a dielectric filled WR90 at 6.7 GHz.

As noted above, the dielectric filling strongly affects the resonant length of the slot, as can be seen by comparison of Fig. 9 with Fig. 5. The average resonant length decreases from about 0.5 λ_0 (empty waveguide) down to ~$0.4\lambda_0$ (filled waveguide) as can also be estimated using (9).

The figure also shows a significant dependence of the resonant length on the slot offset s. This dependence must be taken into account for an accurate design of slot arrays.

IV. CONCLUSIONS

A procedure for the computation of the equivalent admittance of a longitudinal slot in a dielectric filled rectangular waveguide has been presented.

The characterization of the slot is made with the use of the FDTD method, which has shown to be appropriate for this analysis and computational efficiency is achieved using a proper factorization for the equivalent admittance.

The comparison with other methods (MoM) and with experimental results have shown very good agreements with FDTD simulations.

The proposed equivalent circuit allows for a fast and accurate analysis of the radiating slot in a frequency range of the order of 15% and it can be used in the design of large slotted waveguide arrays even when dielectric filling of the waveguides is required to obtain wide scanning angles.

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